

# The Imaginary Time in the Tunneling Process

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(July 25, 1995)

## Abstract

By using techniques developed in quantum cosmology, it is found that a tunneling particle spends purely imaginary time on a barrier region. The *imaginary* time is associated with the stochastic acausal behaviour of a state, while the *real* time is associated with the deterministic causal evolution of a state. For the tunneling case the nonzero imaginary time is associated with the transmission rate of the tunneling process, which is related to the thickness of the barrier. The physical meaning of the zero real time is that the particle instantly jumps from one side of the barrier to the other regardless of the thickness. This leads to the illusion that tunneling particles could actually travel faster than light. The results of recent experiments in quantum optics concerning tunneling time can be thought of as the first experimental confirmation of the existence of imaginary time. Relativity is not violated.

PACS numbers: 98.80.Hw, 98.80.Bp, 05.60.+w, 73.40.Gk

In special relativity it is postulated that light travels in a vacuum at a constant speed (we set the vacuum speed of light  $c$  to be 1) with respect to any inertial frame and nothing can travel faster than light. This postulate leads to causality in relativity. In general relativity, one can always formulate physical laws with respect to local inertial frames at any points of a spacetime. Since in general spacetime is curved, the statements on the speed of light should refer to a local frame only.

In 1932 MacColl [1] argued that a particle tunnels through a barrier without any appreciable delay. Later in 1955, Wigner and Eisenbud [2] claimed that under some circumstances, tunneling particles could travel faster than 1. Recently, experiments in quantum optics by Chiao, Kwiat and Steinberg [3] seem to confirm this. The aim of this paper is to clarify this issue. I want to show that the propagation of light with a speed greater than 1 is just an illusion: it is the effect of the imaginary time spending. Therefore, both relativity and causality remain intact.

Let us begin with the case of light propagation in a transparent dispersive media. This problem was quite thoroughly discussed by Sommerfeld and Brillouin [4]. The phase velocity is equal to  $\omega/k$ , where  $\omega$  and  $k$  are frequency and wave number respectively. The group velocity is defined as  $d\omega/dk$ . It was generally argued that the information propagation should be carried at the group velocity. It was implicitly assumed that the group velocity will never exceed 1 in reality, even theoretically it is quite possible that the group velocity can exceed 1, and at certain frequencies can become infinity, or even negative. However, recent experiments of superluminal propagation show that the group velocity can exceed the speed of light [5]. Therefore, to reexamine the speed of information propagation is inevitable. Another relevant velocity is the so-called energy velocity; it is defined by the ratio of time-averaged Poynting vector and the time-averaged energy density. For transparent and dispersionless media all these velocities are identical.

One can derive the Kramers-Kronig relations of the complex linear susceptibility  $\chi$  solely from the assumption of linearity and causality of wave propagation in the media

$$Re\chi(\omega) = \frac{2}{\pi}P \int_0^\infty \frac{\omega' Im\chi(\omega')}{\omega'^2 - \omega^2} d\omega', \quad (1)$$

$$Im\chi(\omega) = -\frac{2\omega}{\pi}P \int_0^\infty \frac{Re\chi(\omega')}{\omega'^2 - \omega^2} d\omega', \quad (2)$$

where  $P$  denotes the principal value. It is expected that all linear media should satisfy the quite universal Kramers-Kronig relations. If one elaborates the problem on wave propagation by using

the Shannon theory of information, he will find that the information velocity will never exceed 1, even for the case with group velocity  $v_g > 1, \rightarrow \infty$  or  $< 0$ . It means that causality is not violated. One can expect that causality will remain valid even for nonlinear media which does not respect the Kramers-Kronig relations.

The paradox, raised by the experiments performed by Chiao, Kwiat and Steinberg [3] that a tunneling particle seemingly can travel faster than 1, is offered a resolution by these authors as follows [6]: In a typical experiment, the whereabouts of the photon, detected only once, is best predicted by the location of the peak of the wave packet. The wave packet of the tunneling photon gets reshaped, and the peak of the tunneling photon precedes that of a photon traveling unimpeded at the speed of light. They believe that as far as the wavefronts of these photons are concerned, at no point does the tunneling-photon wave packet travel faster than the free-traveling photon.

However, in the opinion of this author, this paradox cannot easily be dispelled this way in the framework of quantum optics. The issue is far more fundamental than it looks. I believe that the issue is just a manifestation of the very nature of time concept.

I would like to present a very simple calculation of time spent by a tunneling particle in the barrier region. In nonrelativistic quantum mechanics, the Schroedinger equation of motion in one dimension for a particle with mass  $m$  takes the form:

$$i\frac{\partial\Psi}{\partial t} = -\left(\frac{\partial^2}{2m\partial x^2} - U(x)\right)\Psi, \quad (3)$$

where  $U$  is the potential of the field and we have set  $\hbar = 1$ . Since the potential is time-independent, one can solve the equation by using the complete set of the stationary states which satisfy

$$\left(\frac{\partial^2\psi}{2m\partial x^2}\right) + [E - U(x)]\psi = 0, \quad (4)$$

where  $E$  is the energy of the stationary states.

In classical mechanics a particle with energy lower than the height of a potential barrier is forbidden to overpass it. In quantum mechanics, it turns out that under this circumstance the particle can tunnel through the barrier, instead. By using the WKB method, one can get the transmission coefficient for the barrier

$$D \approx \exp\left[-2\int_a^b \sqrt{2m(U(x) - E)}dx\right], \quad (5)$$

where the integral is taken for the barrier region  $[a, b]$ , where  $U(x) \geq E$ ,  $a, b$  are the turning points satisfying  $U(a) = U(b) = E$ .

If one turns to the imaginary time regime by setting  $\tau = it$ , then the potential and energy would reverse their signs, and the particle motion can be described as a bounce solution in the potential well. Therefore, the exponent in the transmission coefficient formula can be rewritten as the negative of the action for the classical bounce of the particle in the imaginary time

$$A = \oint p dx, \quad (6)$$

where  $p$  is the momentum of the bounce solution. It was based on this observation that Coleman developed the instanton theory.

The instanton theory has been widely accepted by the particle physics community. In particular, it has fundamental influence on Euclidean quantum gravity and the no-boundary universe. Despite this, people may still consider the above argument merely as a calculation trick in quantum mechanics or quantum field theory. It is my opinion that the current experimental results of quantum optics is a clear confirmation of the physical existence of imaginary time.

The most convenient way to investigate the time problem is the Feynmann path integral approach, as in quantum cosmology [7]. Since in quantum gravity, spacetime itself should be quantized, therefore the time coordinate does not appear explicitly in the path integral. Indeed, a history from an initial closed 3-surface to a final 3-surface is represented by a 4-manifold sandwiched between them, and the time lapse is somehow implied by the manifold. One obtains the wave function by summation of all these histories, and the wave function takes the superposition form of wave packets

$$\Psi \approx \exp iS = \exp[i(S_r + iS_i)], \quad (7)$$

where  $S_r$  and  $S_i$  are the real and imaginary parts of the phase.

It is only after one obtains the wave function, that the time concept will appear explicitly. One can interpret the oscillatory components associated with  $S_r$  of the wave function as classical evolutions in real time and the exponential components associated with  $S_i$  as classical evolutions in imaginary time.

For a system with only one degree of freedom, these two behaviors are mutually exclusive in the configuration space. In the higher dimensional case, in general they are coupled, and these two classical trajectories are mutually perpendicular in the configuration space. One can experience the trajectory in real time as the deterministic and causal evolution. A trajectory in imaginary time

assigns probability  $\propto \exp -2S_i$  to the ensemble of real time trajectories it intersects.

In a closed universe, there does not exist an external time coordinate. What one obtains is the intrinsic time of the universe. For the Hawking massive scalar model [8], time is imaginary in the Euclidean regime and becomes real in the Lorentzian region. In quantum mechanics, in general, the external time is given, and the time derived from the wave function can be identical to the intrinsic time of the particle. Since we are dealing with the issue of how much time a tunneling particle spends on the barrier region, the particle time itself should be quantized, as in quantum cosmology, and one has to adopt the path integral approach. The action from initial position  $x_i$  to final position  $x_o$  is written as

$$I = \int_{x_i}^{x_o} \left[ \frac{m\dot{x}^2}{2} - U(x) \right] dt, \quad (8)$$

where the dot denotes time derivative for the history.

We assume that the potential approached zero on the two far sides of the barrier

$$\lim_{x \rightarrow \pm\infty} U(x) = 0, \quad (9)$$

The wave function for a particle with energy  $E$  initiated at  $x = x_i$  or  $x = x_f$  becomes

$$\psi(x) = \int d[x] \exp(-\bar{I}(x)), \quad (10)$$

the path integral is summed over all trajectories with a fixed momentum corresponding to the energy  $E$  at  $x = x_i$  or  $x = x_o$ . The original path integral is divergent in real time, therefore we have to evaluate it in the Euclidean regime, i.e., we have made the Wick rotation by defining the imaginary time  $\tau = it$  and  $\bar{I} \equiv -iI$  is the Euclidean action.

The path integral over trajectories with a fixed momentum at the right far side of the barrier  $x_o$  ( $x_o > b$ ) represents the stationary state of a particle propagating to the right hand side. The particle from the left is reflected by and penetrated through the barrier. This boundary condition is similar to that in cosmological models. For example, in the Hawking model [8], the derivative of the universe scale with respect to the imaginary time is set to be 1 at the south pole of the Euclidean sector of the spacetime manifold.

The main contribution to the path integral comes from the classical solutions. For the classical solutions, the action becomes

$$I = \int p dx, \quad (11)$$

where  $p$  is the canonical momentum. The classical solutions with real action in real time dominate the wave function outside the barrier, while the bounce solution with imaginary action in imaginary time controls the quantum behaviour within the barrier. Substituting (11) into the path integral, one gets

$$\psi = -\sqrt{\frac{m}{p}} \exp \left[ -\left| \int_a^b p dx \right| + i \int_b^x p dx + \frac{1}{4} i \pi \right], \quad x > b \quad (12)$$

$$\psi = \sqrt{\frac{m}{p}} \exp \left[ -\left| \int_a^b p dx \right| + \left| \int_b^x p dx \right| \right], \quad a < x < b \quad (13)$$

$$\psi = \sqrt{\frac{m}{p}} \exp \left[ i \int_a^x p dx + \frac{1}{4} i \pi \right] + \sqrt{\frac{m}{p}} \exp \left[ -i \int_a^x p dx - \frac{1}{4} i \pi \right], \quad x < a \quad (14)$$

where we have worked the higher order quantum correction by solving the equation (4) through substitution. For simplicity, we have used the approximation that the transmission coefficient  $D \ll 1$ , and the chosen normalization corresponds to the unit probability current density in the incident wave from the left side. For the general case, the solution can be obtained as a superposition of multiple reflections and transmissions at the boundaries of the barrier. It is helpful to note that the particle traveling on the barrier region takes no real time, as we shall show below.

The truncated Schroedinger equation (4) can be thought of as the counterpart of the Wheeler-DeWitt equation in quantum cosmology, in which the time coordinate does not appear explicitly.

To recover the particle time, one can identify

$$p = \frac{\partial S}{\partial x}, \quad (15)$$

and, as expected, the time spent outside (inside) the barrier is real (imaginary), corresponding to the classical (bounce) solution. The imaginary time spent on the barrier region is

$$t = \int_b^a \frac{m dx}{p}. \quad (16)$$

Since both the momentum and time are imaginary within the barrier, then the wave function decays exponentially and it leads to the transmission rate (5).

I believe that this is the simplest way to derive the time the tunneling particle spent on the barrier region. The result is consistent with the Sokolovski and Connor's calculation [9]. Baz' and Rybachenko [10] have proposed the use of the Larmor precession as a clock to measure the time it takes a particle to traverse a barrier. If the particle carries spin 1/2 and a small magnetic field pointing in a direction perpendicular to the spin is confined to the barrier, then the magnetic

field does not actually perform a Larmor precession. Its only effect is to align the spin with the field. The reason is that in the imaginary time, the kinetic energy is split by the Zeeman effect contribution  $\pm w_L/2$ , where  $w_L$  is the Larmor frequency. The energy difference causes the difference of transmission rates. It results experimentally in a spin components  $w_L|t|/2$  along the magnetic field. This method has been used to obtain highly polarized electrons from metals coated with a thin film of a ferromagnetic semiconductor.

In phenomenology, light propagation along a media can be described by a massless particle in a potential. The wave function obeys the Klein-Gordon equation. One can solve the stationary state equation and use the eigenstates with positive energy only. The above argument should remain valid. The potential should be semi-positive-definite, then the velocity of photons should not exceed 1 in real time and it will spend imaginary time on the barrier region.

The key issue is to interpret the meaning of the imaginary time. From the viewpoint of the intrinsic time of the particle, it experiences a real time as traveling from the left. After entering the barrier it experiences an imaginary time, its behaviour is no longer deterministic and causal. Even for a hermitian Hamiltonian, the evolution along imaginary time becomes nonunitary. In our case, the wave function decays exponentially as shown by eq. (13). It takes no real time to pass the barrier and emits from the right turning point instantly, and then resumes the lorentzian evolution. The reason that an outside observer can only sense the real time lapse, is that all observations and human beings' consciousness are connected with causal and deterministic elements of any phenomena. The imaginary time can manifest itself through some stochastic behaviour.

From the viewpoint of the extrinsic time, the evolution of the whole system is unitary. Even the probability of a particle tunneling is less than 1, the total probability of tunneling and reflecting remains 1.

We experience imaginary time daily. Maybe all stochastic phenomena in Nature are the manifestation of imaginary time. It is well known that if one turns to imaginary time from real time, then a path integral in quantum mechanics becomes a partition function in statistical physics. Indeed, imaginary time has become common sense in black hole physics and quantum cosmology. The fact that a tunneling particle decays on barrier region is well understood by using imaginary time. A photon tunneling through the barrier instantly regardless of its thickness causes the illusion that light can travel faster than 1. Locally, a photon always travel with speed 1 in a vacuum. In the

barrier region, it travels along imaginary time. All these are not only consistent with relativity, but can also be thought of as a toy version of traveling from one universe to another, or from one region to another of the same universe through a wormhole. General relativity seems to offer the possibility of this kind of rapid intergalactic travel.

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